If the degrees of all the vertices in a planar graph are 5 or higher, then the smallest possible number of vertices with degree 5 is 12.

Proof:

Let V, E, and R be the number of vertices, the number of edges, and the number of regions (including the outer region) in the graph, respectively. By Euler's formula, $V - E + R = 2$. Without any loss of generality we can assume that the graph is fully triangulated — otherwise we just add edges. Then $E = \frac{3R}{2}$ because each region has three edges and each edge belongs to two regions. So $R = \frac{2E}{3} \Rightarrow V - E + R = V - E + \frac{2E}{3} = V - \frac{E}{3} = 2 \Rightarrow 6V - 2E = 12$. Let d_v be the degree of the vertex v. Then $E = \frac{1}{2}\sum_v d_v$, because d_v edges come out of v and each edge connects two vertices. $2E = \sum_v d_v \Rightarrow \sum_v (6 - d_v) = 6V - \sum_v d_v = 6V - 2E = 12$. Since $d_v \geq 5$, $6 - d_v \leq 1$. At least 12 of the addends in $\sum_v (6 - d_v)$ must be equal to 1 for the sum to be 12, so at least 12 of the d_v must be equal to 5.

A flattened icosahedron is an example of a graph with 12 vertices of degree 5.